

RATIONAL INCOMPETENCE OF VOTERS

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THE ESSENTIAL feature of democracy is that all eligible citizens in a society have a right to participate in the collective decision-making process.¹ The efficacy of democracy therefore depends on how citizens vote (and otherwise participate) in this process. Presumably (or ideally, at least), ordinary citizens vote for the candidate or policy they consider best on the basis of social circumstances and information gathered by themselves, but the question is how reliable they (collectively) are.

Epistemic democracy is a philosophical perspective that justifies democracy on the grounds of its ability to make good/correct decisions.² The Condorcet jury theorem (CJT) is the central mathematical theorem for epistemic democracy.³ According to the CJT, decision-making by majority selects the right outcome/policy if citizens decide their votes independently, if each voter is competent to a certain degree, and if the number of citizens involved in the decision-making process is large enough. This theorem has been applied in various ways and contexts and lends support to the idea of epistemic democracy.⁴ However, the CJT has been criticized as well. Criticisms are linked to the formal conditions assumed by the CJT. A classical criticism puts the *independence condition*, which requires that the votes of citizens are independent, in doubt. If there is something that causes a strong correlation between votes, the CJT fails.⁵ A significant number of studies have considered how such correlation can occur.⁶

- 1 It is common that individuals must be above a certain age in order to be eligible to vote. In some countries, people convicted of certain crimes may be prohibited from voting.
- 2 Anderson, "The Epistemology of Democracy," 10–15.
- 3 For important early work on the CJT, see Black, *The Theory of Committees and Elections*, 159–78; Grofman, "A Comment on 'Democratic Theory,'" 100; and List and Goodin, "Epistemic Democracy," 285.
- 4 Goodin and Spiekerman, *An Epistemic Theory of Democracy*.
- 5 The CJT holds under some types/levels of correlation. Berg, "Condorcet's Jury Theorem, Dependency Among Jurors," 91–92; Ladha, "Condorcet's Jury Theorem in Light of de Finetti's Theorem," 77–82; and Pivato, "Epistemic Democracy with Correlated Voters," 59–61.
- 6 See, for example, Boland, "Majority Systems and the Condorcet Jury Theorem," 185–86; Boland et al., "Modelling Dependence in Simple and Indirect Majority Systems," 83–86;

A relatively new and quite radical criticism concerns the *competence condition*, according to which each voter votes for a right option with a certain probability p that is greater than half. The competence condition assumes that each voter's decision is better than random. In regard to this condition, some critics have argued that citizens are unreasonable, ignorant, and therefore incompetent (i.e., worse than random). For example, Brennan builds his case for epistocracy (decision-making by experts) on this incompetency.⁷

In this paper, I examine the possibility of incompetency among reasonable and non-ignorant agents.⁸ I characterize the properties of such agents using the framework of Bayesian rationality. That is, I assume that each citizen updates their belief(s) based on Bayesian inference after getting new, *nonmisleading* information and that each citizen maximizes their expected utility. I show that even with this assumption, there are realistic cases in which agents are incompetent (i.e., the competence condition is violated). Notably, this is a new possibility of voter incompetency, given that unreasonableness or ignorance are the source(s) of voter incompetency in existing work. Importantly, Bayesian voters are not subject to cognitive bias; the possibility of misleading information is excluded.⁹ My results demonstrate that the *asymmetry of signals* is a threat to democracy. That said, the asymmetry of signals also disrupts decision-making by epistocrats; thus, it can also be a threat to epistocracy. Moreover, I offer a single example to illustrate that epistocracy can be (conditionally) worse than democracy, based on the asymmetry of signals.

Cato and Inoue, "Are Good Leaders Truly Good?" 441–42; and Estlund, "Opinion Leaders, Independence, and Condorcet's Jury Theorem," 138–39.

7 Brennan, *Against Democracy*.

8 Bayesian rational agents satisfy the features of Brennan's "vulcans," who are the ideal types of experts. Brennan writes:

Vulcans are perfectly rational. An ignorant vulcan would know they are ignorant, and thus would be almost entirely agnostic about political issues. If they decided to learn more, they would seek out information from credible sources. They would conform their beliefs to the best available evidence. A vulcan would look not merely at evidence in favor of different views but also evidence against these views. They would change their minds whenever the evidence called for it. They would consult peers and take disagreement seriously, and would gladly accept criticism, since they want to avoid error. "Thanks for correcting me and pointing out my mistakes!" They would hold beliefs only as strongly as the evidence allows. (*Against Democracy*, 36–37)

9 Brennan mentions two ideal types of incompetent voters: "hobbits" and "hooligans" (*Against Democracy*, 1–22). The former includes voters who do not care about social issues, while the latter includes voters with a strong cognitive bias. Bayesian voters are neither of the two.

This discussion note consists of two main sections. The first shows that Bayesian voters are competent if signals are symmetric; therefore, the CJT works. The second section shows that if signals are asymmetric in a certain way, the same voters can be incompetent, and thus, the CJT fails.

1. COMPETENCE OF BAYESIAN AGENTS UNDER SYMMETRIC SIGNALS

Let us assume that there are two states: L and R . Each state occurs with a probability 0.5, but voting citizens do not know which state they are actually in. Voters get some signal that represents a relevant piece of information, and there are two such signals, A and B . Voters independently receive one of these signals as follows:

- If the true state is L , each voter gets signal A with probability p and signal B with probability $1 - p$;
- If the true state is R , each voter gets signal A with probability $1 - p$ and signal B with probability p .

We assume that p is not equal to 0.5.¹⁰ The probability distribution is shown in table 1. Each voter is assumed to vote for either of the two policies L or R . The policy that receives the largest number of votes is collectively selected. If the selected policy coincides with the true state (that is, the right policy is selected), then each voter obtains one unit of utility. Otherwise, they obtain zero. We assume sincere voting and that voters do not have any predetermined ideological bias, so each voter chooses a policy (L or R) to maximize their expected utility.¹¹ I assume that this prior probability structure is commonly known among agents; moreover, the prior distribution is assumed to be correct, and there is no possibility of misleading information.

Table 1. Symmetric Probability Structure

	State L	State R
Signal A	p	$1 - p$
Signal B	$1 - p$	p

10 If this assumption is not satisfied (i.e., $p = 0.5$), then the signals provide no information, which would be essentially the same as no signal.

11 This means that we ignore the possibility of strategic voting, which is examined by Austen-Smith and Banks, "Information, Aggregation, Rationality, and the Condorcet Jury Theorem," 34–35.

It is assumed that each agent knows the probability structure of signals. In a situation like this, voters who apply Bayesian inference update their beliefs about states (and thus about the right policies) after receiving their signals. If an agent obtains A as a signal, they update their belief by applying Bayes's formula:

$$\begin{aligned} P(L | A) &= \frac{P(A | L)P(L)}{P(A)} \\ &= p. \end{aligned}$$

Analogically, $P(R | A) = 1 - p$. This means that if a voter gets signal A , then their subjective belief that L is the true state becomes p and that R is the true state becomes $1 - p$. Note that the expected utility of voting for L is p , while that of voting for R is $1 - p$. Therefore, if p is larger than 0.5 (resp. p is smaller than 0.5), then voters who receive A vote for L (resp. R).

The case with signal B is derived as follows:

$$P(L | B) = 1 - p \text{ and } P(R | B) = p.$$

If p is larger than 0.5 (resp. p is smaller than 0.5), then voters who receive B vote for R (resp. L).

Note that each agent is competent in the sense that they vote for the right policy with a probability that is higher than 0.5 in each state. Now assume that p is greater than 0.5 . If L is the actual state, then the probability of getting A as a signal is higher than 0.5 . Because a voter votes for L when A is received, the probability that they are correct is higher than 0.5 from the *ex ante* viewpoint. If the actual state is R , then the probability of getting B as a signal is larger than 0.5 , and again, the probability that they are correct is higher than 0.5 . In summary, voters are competent in either state. This is true for any value of p as long as p is not 0.5 . To see this, it suffices to consider the case where p is smaller than 0.5 . In state L , the probability of getting B is higher than 0.5 . Then, the voter votes for L when B is received, and the probability that they are correct is higher than 0.5 . An analogical argument holds in state R . Each voter is competent when p is smaller than 0.5 .

2. RATIONAL INCOMPETENCE AND ASYMMETRIC SIGNALS

Extending the argument from the previous section, I show in this section how a Bayesian rational agent can be incompetent (even in the absence of misleading information). Significantly, the model in the previous section assumes that the probability distribution for signals is symmetric. Specifically, the probability

of getting *A* under *L* is equal to that of *B* under *R*. (See table 1 again.) The voter competence shown in the previous section depends on this feature.

Now let us consider the following (potentially) asymmetric probability distribution.

- If the true state is *L*, each agent gets signal *A* with probability *p* and signal *B* with probability $1 - p$;
- If the true state is *R*, each agent gets signal *A* with probability $1 - q$ and signal *B* with probability *q*.

The probability distribution is shown in table 2. Note that this table coincides with table 1 if *q* is equal to *p*. However, we assume that *q* is not equal to *p*.

Table 2. General Structure of Probability Distributions

	State <i>L</i>	State <i>R</i>
Signal <i>A</i>	<i>p</i>	$1 - q$
Signal <i>B</i>	$1 - p$	<i>q</i>

If an agent receives signal *A*, they update their belief in the following manner:

$$\begin{aligned}
 P(L | A) &= \frac{P(A | L)P(L)}{P(A)} \\
 &= \frac{p}{p + (1 - q)}.
 \end{aligned}$$

Analogously, $P(R | A) = (1 - q) / (p + (1 - q))$. Note that the expected utility of voting for *L* is $p / (p + (1 - q))$, while that of voting for *R* is $(1 - q) / (p + (1 - q))$. Therefore, if *p* is larger than $1 - q$ (resp. $1 - q$ is larger than *p*), then voters who receive *A* vote for *L* (resp. *R*).

The case with signal *B* is derived as follows:

$$P(L | B) = \frac{1 - p}{(1 - p) + q} \text{ and } P(R | B) = \frac{q}{(1 - p) + q}.$$

If $1 - p$ is larger than *q* (resp. *q* is larger than $1 - p$), then voters who receive *B* vote for *L* (resp. *R*).

Consider the probability distribution in table 3. According to the voting patterns described above, an agent votes for *L* if they receive *A*, while they vote for *R* if they get signal *B*. In state *L*, agents cannot be competent on average even though they update their belief based on Bayesian inference. Notably, each

agent receives signal *A* with probability 0.3 and *B* with probability 0.7. If a voter receives *A*, then they vote for *L*, which is correct. However, if they receive *B*, then they vote for *R*, which is not correct. This means that the voters are correct with probability 0.3; they vote for the wrong policy with probability 0.7. Hence, the voters are incompetent in state *L*. (Note that voters are correct with probability 0.9 in state *R*.) The CJT fails because of this incompetence.

Table 3. *Asymmetric Probability Distributions:
Rational Incompetence*

	State <i>L</i>	State <i>R</i>
Signal <i>A</i>	0.3	0.1
Signal <i>B</i>	0.7	0.9

To see that the CJT fails, assume that the number of voters is very large. The share of voters who vote for *L* approximates 0.3 (resp. 0.1) in state *L* (resp. *R*). The share of voters who vote for *R* approximates 0.7 (resp. 0.9) in state *L* (resp. *R*). Hence, it is effectively guaranteed that the wrong policy is selected in state *L*. (On the other hand, it is similarly certain that the correct policy is selected in state *R*.)

It must be emphasized that all of these voters are rational because they follow Bayesian reasoning and maximize their expected utility based on their updated beliefs. Despite this rationality, the probability that a voter's choice is right can be lower than 0.5 in some state. Certainly, Bayesian rational agents can easily hold wrong beliefs when there is sufficient misleading evidence. However, in this case, rational agents tend to fail even if there is neither misperception nor misleading information involved. We can call this phenomenon *rational incompetence*. The cause of this rational incompetence is the probability structure of signals. To be precise, the information each person gets is asymmetric toward *B* (i.e., the probability of getting *B* is higher than 0.5 in both states). This shows that if information in society is unbalanced in a certain way, the competence of votes is compromised even if they are all rational, which can be a threat to (epistemic) democracy.

Notice that this rational incompetence is completely different from rational ignorance, which is well known in the literature on voting. A voter is rationally ignorant when they do not (sufficiently) inform themselves about a political issue because of the cost of obtaining information. In the case of rational incompetence, on the other hand, voters are eager to obtain relevant information and accurately process the information they get. Hence, voters are far from ignorant in such cases.

It is important to understand that asymmetric signals (as in table 3) are not a mere theoretical possibility. On the contrary, such signals are not especially extreme and can be quite realistic. To illustrate, consider a case in which someone who has a fever wants to know whether they have COVID-19. This person will use their symptoms as signals. Here, the prior probability of COVID-19 is assumed to be 50 percent. Assume that 10 percent of people who are infected with COVID-19 experience a new loss of taste or smell; this symptom occurs in less than 1 percent of fever cases that are not caused by COVID-19. If someone experiences a loss of taste or smell, they will believe that there is a high chance of having COVID-19. While this updating is completely rational, the signal structure underlying it is strongly asymmetric, as shown in table 4.

Table 4. COVID-19 and Its Symptoms

	COVID-19	Not COVID-19
Loss of taste or smell	0.10	0.01
No loss	0.90	0.99

As another example, consider policymaking associated with nuclear power plants. Nuclear power plants should not be built in areas where there is a high risk of earthquakes. Even in a high-risk area, the probability of the occurrence of a very strong earthquake within the next fifty years is extremely low. (The probability is even smaller outside the high-risk area.) When past earthquake frequency is used as a signal, signal structures are strongly asymmetric. Although earthquakes are not independent signals for voters, it is not unrealistic to assume that a similar asymmetry exists in the case of independent signals such as those considered in the CJT.

Furthermore, the theoretical mechanism for rational incompetence also works if the number of signals is smaller than the number of states. Assume, for example, that there are only two signals, *A* and *B*, but three states, *L*, *M*, and *R*, such that each state occurs with probability $1/3$. The probability distribution of the two signals in the three states is shown in table 5. Under this probability distribution, each agent votes for *L* if they receive signal *A*, while they vote for

Table 5. Three States and Two Signals:
Rational Incompetence Due to Signal Insufficiency

	State <i>L</i>	State <i>M</i>	State <i>R</i>
Signal <i>A</i>	0.7	0.6	0.3
Signal <i>B</i>	0.3	0.4	0.7

R if they get *B*. Hence, no one votes for *M*, even when *M* is the actual state. This implies that all voters will be completely incompetent in *M*; therefore, the CJT fails. Arguably, it is likely that the problem of asymmetry becomes more severe when there are more states.

Lastly, the same rational incompetence can also affect epistocracy. In *restricted suffrage*, which has been suggested by advocates of epistocracy, only a finite number of qualified experts will vote.¹² Presumably, these experts will be rational and will use the knowledge they have gained to decide on the options they can vote for. However, if there are asymmetries in the signals, then there is a good chance that their rational voting will not increase the probability that the right policy is chosen. Hence, epistocracy may not work as well as assumed by its advocates. Notably, in case of asymmetric signals, the rationality of agents does not help to increase the probability of choosing the right policy; thus, restricting voting rights on grounds of irrationality is not a plausible method to assure the best outcome.

Next let us examine a scenario in which epistocracy does not have a strict competency advantage over democracy. In the situation described in table 3, I assume that experts vote based on Bayesian updating, while nonexperts vote randomly without any deliberate thought. In other words, experts vote for *L* when they receive signal *A* and for *R* when they receive signal *B*. By contrast, nonexperts vote for either option with equal probability.¹³ In this scenario, the CJT fails for both democracy and epistocracy, but the failure manifests differently in each regime. In the case of epistocracy, the majority believes that *R* is likely to be true, regardless of the true state; therefore, an incorrect outcome tends to be selected when the true state is *L*. By contrast, under democracy, where nonexperts vote randomly, each outcome is selected with probability 0.5, regardless of the true state. Consequently, when the true state is *L*, democracy is more likely to yield the correct outcome compared with epistocracy. Thus, one could argue that democracy is *conditionally* better than epistocracy when the true state is *L*.¹⁴

3. CONCLUDING REMARKS

This discussion note has shown that people can easily become incompetent voters depending on the probability distribution of signals (i.e., information).

12 See Brennan, *Against Democracy*, 213–18.

13 This assumption aligns with Brennan's concept of "hobbits." See note 9 above.

14 Notably, this does not imply that epistocracy is unconditionally worse than democracy. Both regimes have an unconditional (or *ex ante*) probability of 0.5 for making the correct choice.

This paper's contribution is not only that it demonstrates the possibility that the competency condition is not satisfied but also that it identifies how it is not satisfied. Importantly, my analysis is not based on a simple observation that Bayesian rational agents believe the wrong information if they receive sufficient misleading evidence. I demonstrate that the competency condition may fail even if an agent gets the right information (i.e., that a loss of taste is a sign of COVID-19). This situation is likely to occur when signal availability is limited. Hence, what the CJT actually requires is that sufficient informational signals are available for all options and possibilities.

The information structure identified in the paper is indeed likely to occur in real-world scenarios. For instance, consider a scenario in which a candidate might have dementia. Even if this were the case, the likelihood of voters receiving a clear signal indicating dementia is low. Generally, signals suggesting a candidate's unsuitability or highlighting fundamental policy errors are often not easily accessible, even to experts. This leads to an asymmetric information structure. Such asymmetries can result in voters making wrong decisions, thereby significantly impacting the democratic process. It is crucial to understand and acknowledge the prevalence of these information asymmetries to ensure a nuanced debate on the merits and challenges of both epistocratic and democratic systems.¹⁵

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