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THE VALUE OF A LIFE-YEAR AND THE INTUITION OF UNIVERSALITY

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As highlighted by Sen in his early criticism of national accounts statistics, the measurement of the achievements of a society can hardly abstract from how long the members of that society live.[1] Individual lifetime is a key dimension to be taken into account in the measurement of social achievements when one wants to measure the degree of development of a society or social welfare. That intuition motivated the inclusion of indicators of human lifetime—such as period life expectancy at birth—in the construction of indexes of development, such as the Human Development Index (HDI), and in the construction of inclusive indexes of well-being, such as equivalent income.[2] Moreover, since various public policies modify the production and the distribution of life-years within a population (e.g., health care programs, transportation policies, and environmental policies), the lifetime dimension can hardly be ignored when considering the design of policies.[3]

Taking human lifetime into account requires the social evaluator or the government to find a way to weight the quantity of life-years against other relevant dimensions of life. In other terms, the social evaluator needs to find a way to assign a value to life-years. Assigning a value to life-years is necessary not only for the measurement of human development or well-being, but also to be able to solve policy dilemmas involving various implications in terms of the production and distribution of life-years within a population. This necessity to assign a value to life-years has given rise in economics to the increasingly large literature on the value of a statistical life.[4] Empirical estimates of the value of a statistical life

1 See Sen, "On the Development of Basic Income Indicators to Supplement GNP Measures."

2 See United Nations Development Programme, *Human Development Report 1990* and *Human Development Report 2010*. On the equivalent income index, see Fleurbaey and Ponthiere, "Prevention against Equality?"

3 See Sen, "Mortality as an Indicator of Economic Success and Failure"; Broome, *Ethics out of Economics* and *Weighing Lives*.

4 See Jones-Lee, *The Economics of Safety and Physical Risk*. The value of a statistical life is

have become key parameters in cost-benefit analyses, as well as in the analysis of optimal policies, in particular in the context of climate change.

The empirical literature on the value of a statistical life has a purely positive nature: it aims at quantifying how individuals tend, in real life, to solve trade-offs involving risk about the duration of life, or in other words, how individuals are willing to exchange money against variations of the risk of death. Empirical studies of various kinds (wage-risk studies, contingent valuation methods, etc.) show that the value of a statistical life varies with several factors, such as age and occupation, and increases with income. For instance, in his meta-analysis of empirical studies, Miller provides a rule of thumb for the valuation of a life, which is a simple linear transformation of the gross domestic product (GDP) per capita.[5]

Obviously, it does not logically follow from those empirical studies that, when measuring social achievements or designing policy objectives, the social evaluator should make the valuation of life or the valuation of a life-year dependent on income or on any of those factors. Empirical studies on the value of life may well show that the value of a life-year is increasing with income, but such a positive premise cannot, if taken separately, lead to any normative corollary. Drawing such a conclusion would be nothing other than an occurrence of a naturalistic fallacy. There is thus a need to examine, at the normative level, how a social evaluator should value an extra life-year—that is, the principles that should govern such a valuation of life-years.

From a normative perspective, the valuation of a life-year leads to two conflicting intuitions: on the one hand, the intuition of universality, and on the other hand, the intuition of complementarity.

According to the intuition of universality, the value of a life-year should be universal. The value of a life-year should be the same whatever contexts are considered; in particular, a life-year should have exactly the same value when we consider a poor or rich country. Thus, from that perspective, the fact that life-years are more valued in richer countries than in poorer countries should be irrelevant when considering the social valuation of those life-years: universalism requires life-years to be valued in the same way, independently from the context, and, in particular, independently from the associated quality of life.

Such an intuition of universality concerning life-year valuations was defended, among others, by Anand, who criticized the new HDI—which is based on a geometric average rather than an arithmetic average across income, lifetime, and

defined as the value that x individuals assign to a reduction of the risk of death from $1/x$ to 0, leading to saving one life.

5 See Miller, "Variations between Countries in Values of Statistical Life." According to Miller, the value of a statistical life lies between 120 and 180 times the GDP per capita.

education dimensions—on the grounds that it violates this intuition of universal valuation of life-years.[6] According to Anand, the value of an extra life-year should be the same, whatever one considers a rich or poor country. That intuition of universality is satisfied by the standard HDI, but not by the multiplicative HDI.

This intuition of universality conflicts with another intuition, which can be called the intuition of complementarity. According to that intuition, the quantity of life cannot be valued independently from the quality of life. The reason lies in the singular nature of lifetime in comparison to other goods. Lifetime is not a good like a standard good, which could be enjoyed on its own. On the contrary, lifetime is like a "container," whose value depends on what it will allow for—that is, on what lies "within the container" (life events, activities, projects, etc.). As a consequence, the valuation of life-years cannot be made independently from the associated quality of life. A corollary of this is that, when the quality of life varies, the value of the quantity of life cannot remain the same, and, hence, cannot be universal, in opposition to what the intuition of universality recommends.

The intuition of complementary can take two forms: a weak version, according to which the value of a life-year depends, among other things, on the quality of that life-year, and a strong version, according to which the value of a life-year depends only on the quality of that life-year. Although the latter version is much stronger than the former, it finds some support in several simple thought experiments. For instance, if one could artificially slow down life without modifying the number of life events that take place in that life, then one could hardly regard this lengthening of life as valuable: enlarging the size of the "container" without modifying its contents could hardly make a life better. Alternatively, consider another thought experiment, where one can shut down life during one hour, and shorten all lives by that amount, without anyone being aware of that shutdown. As long as this temporary shutdown was unnoticed, and did not affect events among humans, it is hard to see how this reduction of the size of the "container" could reduce the value of life. Thought experiments provide some support for the strong version of the intuition of complementarity. But it is important to stress that even the weak version of that intuition is in conflict with the intuition of universality.

In order to better present the differences between the intuition of universality and the intuition of complementarity, it can also be useful to refer to the concept of intrinsic value of life, that is, the value of life *per se*, independently from the characteristics of that life. According to the intuition of universality, the value of a life-year is composed exclusively of its intrinsic value, and thus

6 See Anand, "Recasting Human Development Measures." The treatment of lifetime within the multiplicative HDI is also criticized by Ravallion; see "Troubling Trade-offs in the Human Development Index."

does not depend on the quality of that life-year (only the "container" has value). On the contrary, the weak version of the intuition of complementarity states that the value of a life-year may or may not have an intrinsic component, but for sure includes a component that is related to the quality of that life-year (what is inside the "container" matters, and possibly the "container" as well). Finally, the strong version of the intuition of complementarity states that the value of a life-year only includes a component that is related to its quality, and includes no intrinsic value component (only what is inside the "container" is valued, not the "container" itself).

The incompatibility between the intuition of universality and the intuition of complementarity raises deep challenges for the valuation of life-years. Although this is not often explicitly acknowledged, a large part of the literature on the valuation of life relies on the intuition of complementarity, and as such violates the intuition of universality.[7] Those violations are problematic only to the extent that the intuition of universality is worth being pursued. But the ethical appeal of the intuition of universality is hard to evaluate, simply because no precise account of that intuition has been given so far. As a consequence, it is difficult to have an idea of the precise implications of adopting such an intuition.

The goal of this paper is precisely to provide a more accurate account of the intuition of universality, in order to better discuss its implications for the valuation of life-years. In order to better understand what the intuition of universality is, we propose to study its implications for the valuation of life-years when that intuition is true. For that purpose, this paper will proceed in two stages. In a first stage, we provide three distinct definitions of the intuition of universality, in terms of the constraints this intuition imposes on the form of value functions aimed at valuing lives. That first approach informs us about the formal constraints that the intuition of universality imposes on the valuation of the quantity and quality of life, but does not inform us about the priority to be given in allocation problems. Then, in a second stage, we consider a more general approach, in terms of social preferences, and reformulate those three accounts of the intuition of universality, in order to explore their consequences in terms of priority when considering problems of life-year allocations.

Our main results are twofold. First, we show that the three distinct accounts

7 This is true for indicators of well-being relying on a life-cycle perspective, such as the equivalent income approach. See Costa and Steckel, "Long-Term Trends in Health, Welfare, and Economic Growth in the United States"; Nordhaus, "The Health of Nations"; Becker, Philipson, and Soares, "The Quantity and Quality of Life and the Evolution of World Inequality"; Fleurbaey and Blanchet, *Beyond GDP*. This is also the case for normative studies on compensation for unequal lifetime, such as Fleurbaey and Ponthiere, "Prevention against Equality?"

of the intuition of universality lead to quite counterintuitive implications from a normative perspective. One of these is shown to be in conflict with a basic property of monotonicity, whereas two other accounts of the intuition of universality lead to indifference with respect to how life-years are distributed within the population, which is also quite counterintuitive. Thus adopting a universal perspective on life-years valuations—and, thus, abstracting from the associated quality of those life-years—leads to quite questionable consequences. Those negative results support the abandonment of the intuition of universality. However, we show that abandoning the intuition of universality, and adopting instead the intuition of complementarity, does not prevent a social evaluator from giving priority, when allocating life-years, to individuals with the lowest quality of life.

This paper is related to several branches of the literature. First of all, it is related to the literature on the measurement of human development, such as Ravallion and Anand, who criticize the geometric HDI on the grounds of an intuition that is strongly related to the intuition of universality as explored in this paper.[8] Our paper is also related to multidimensional indicators of well-being inclusive of the lifetime dimension, such as the equivalent income.[9] Second, this paper is also related to the literature on the normative foundations of the valuation of life.[10] Actually, there is a formal similarity between some arguments developed by Broome against the intuition of neutrality in the context of valuing the life of a person, and some of our arguments against the intuition of universality in the context of valuing a life-year.[11] Third, this paper is also related to the normative literature on fairness in the context of life and death.[12] The design of optimal policies is not independent from how lives are valued. This makes the distinction between the intuition of universality and the intuition of complementarity most relevant for policy purposes.

The rest of the paper is organized as follows. Section 1 presents three distinct accounts of the intuition of universality and explores their consequences on the structural form of value functions. Section 2 develops an approach based

8 See Ravallion, "Troubling Trade-offs in the Human Development Index"; and Anand, "Recasting Human Development Measures."

9 See Costa and Steckel, "Long-Term Trends in Health, Welfare, and Economic Growth in the United States." See also Nordhaus, "The Health of Nations"; Becker, Philipson, and Soares, "The Quantity and Quality of Life and the Evolution of World Inequality"; Fleurbaey and Blanchet, *Beyond GDP*.

10 See Broome, *Ethics out of Economics* and *Weighing Lives*.

11 See Broome, *Weighing Lives*.

12 See Fleurbaey and Ponthiere, "Prevention against Equality?" See also Adler, Hammitt, and Treich, "The Social Value of Mortality Risk Reduction."

on social preference orderings, reformulates the three accounts of the intuition of universality in that framework, and explores their consequences for the social ranking of distributions along the quantity and quality of life dimensions. Section 3 proposes simple ways to reconcile the abandonment of the intuition of universality with the priority given to individuals with low qualities of life in problems of life-years allocation. Concluding remarks are in section 4.

1. THREE ACCOUNTS OF THE INTUITION OF UNIVERSALITY: AN AXIOLOGICAL APPROACH

According to the intuition of universality, the valuation of a life-year is independent from the context under study, and thus independent from the associated quality of life. That intuition is in need of a more exact formulation, since the independence to which it refers may take various distinct forms. In this section, we propose three distinct accounts of the intuition of universality.

For that purpose, we adopt an axiological approach: our main object of study is a value function $V(.)$, which is defined on a life, which is itself defined as a vector $(a, b, \ldots, k)$ whose entries $a, b, \ldots, k$ correspond to the quality of each life-year. This section explores the formal constraints that the intuition of universality imposes on the structure of the value function $V(.)$. As such, this constitutes a first step toward a better understanding of the intuition of universality.

At the very outset, it should be stressed that a first, basic reading of the intuition of universality consists of stating that the value of a given life $(x, \ldots, x)$ in a country C should be exactly equal to the value of the same life in another country C'. That basic conception of the universality of the valuation of life is presented below, as the intuition U_0.

Intuition of Universality (U_0):

$$V((x, \ldots, x)_{country\ C}) = V((x, \ldots, x)_{country\ C'})$$

The formulation U_0 of the intuition of universality is quite intuitive: it makes a lot of sense to assume that the value of a life does not depend on the country where that life takes place. U_0 is thus a quite intuitive property that a value function $V(.)$ should satisfy.

Note, however, that the property U_0 is not really original, because it coincides merely with a standard anonymity condition. Anonymity being widely used in social evaluations—without an explicit reference to the intuition of universality—we believe that the property U_0 does not exhaust what the intuition of universality is about, and, in some sense, does not suffice to do justice to the

intuition of universality. Therefore we will, in the rest of this paper, take U_0 for granted—but as an anonymity condition—and explore the consequences of alternative formulations of the intuition of universality.

The intuition of universality can be formulated in terms of the variation of value induced by the addition of an extra life-year. One possible formulation of the intuition of universality consists of stating that the variation of value associated with the addition of an extra life-year should depend neither on the quality of the added life-year, nor on the quality of previous life-years, nor on the number of previous life periods (duration of initial lives). More formally, that formulation is:.

Intuition of Universality (U_1):

$$V((a, \ldots, g) + x) - V((a, \ldots, g)) = V((h, \ldots, m) + y) - V((h, \ldots, m))$$

The left-hand side of the above equality is the variation in value when a life-year with a quality x is added to a life of quality $(a, \ldots, g)$, whereas the right-hand side is the variation in value when a life-year with quality y is added to a life of quality $(h, \ldots, m)$.

The formulation U_1 of the intuition of universality states that the value of a life-year is universal, in the sense that it involves a *triple independence*: (i) independence with respect to the quality of the added life-year; (ii) independence with respect to the quality of previous life-years; (iii) independence with respect to the duration of initial lives to which a life-year is added (since the left-hand side and the right-hand side of the above equality may involve initial lives of unequal lengths). As such, the formulation U_1 captures a strong conception of universality.

To see how strong that conception of universality is, let us take the case of the addition of a life-year either in the United States, where average standard of living and life expectancy are high, or, alternatively, in Ghana, where standard of living and life-expectancy are lower. The condition U_1 states that the addition of a life-year in the United States (with US standard of living) has exactly the same value as the addition of a life-year in Ghana (with Ghana's standard of living). That condition states also that adding an extra life-year in Ghana with US standard of living has the same value as adding an extra life-year in the US with Ghana's standard of living.

Although it may seem appealing at first glance, the formulation U_1 of the intuition of universality has implications that are not attractive. In particular, the formulation U_1 conflicts with the monotonicity condition, stating that the value of a life increases with its quality. To see that conflict, let us take, here again, the example of the addition of a life-year in the United States or in Ghana, with

either US life quality (equal to 2) or Ghana's quality (equal to 1). Here are four alternative options:

A. Adding a life-year in Ghana with Ghana's standard of living
B. Adding a life-year in the US with the US's standard of living
C. Adding a life-year in Ghana with the US's standard of living
D. Adding a life-year in the US with Ghana's standard of living

The conception $U1$ of the intuition of universality implies that the value assigned to the addition of a life-year in case A must be equal to the value assigned to the addition of a life-year in case B, that is:

$$V((1,\ldots,1)+1)-V((1,\ldots,1))=V((2,\ldots,2)+2)-V((2,\ldots,2)).$$

However, by monotonicity, we have also that the value assigned to the addition of a life-year in case C exceeds the value assigned to the addition of a life-year in case A (where the added life-year has a lower quality, while everything else is left unchanged), that is:

$$V((1,\ldots,1)+2)-V((1,\ldots,1))>V((1,\ldots,1)+1)-V((1,\ldots,1)).$$

Still by monotonicity, we have also that the value assigned to the addition of a life-year in case B exceeds the value assigned to the addition of a life-year in case D (where, again, the added life-year has a lower quality, everything else remaining unchanged), that is:

$$V((2,\ldots,2)+2)-V((2,\ldots,2))>V((2,\ldots,2)+1)-V((2,\ldots,2)).$$

Given that $V((1,\ldots,1)+2)>V((1,\ldots,1)+1)$, and $V((2,\ldots,2)+2)>V((2,\ldots,2)+1)$, one obtains, from the first equality, the following inequality:

$$V((1,\ldots,1)+2)-V((1,\ldots,1)>V((2,\ldots,2)+1)-V((2,\ldots,2)).$$

This inequality means that adding a life-year with US standards in Ghana leads to a higher gain in value compared to adding a life-year with Ghana's standards in the US.

That inequality is in contradiction with the formulation $U1$ of the intuition of universality. According to that conception of universality, for sure a life-year with US standards in Ghana should be *equally good* to adding a life-year with Ghana's standards in the US. Hence, we reach here a contradiction, which implies that the formulation $U1$ of universality is not logically compatible with the monotonicity condition.

That proof by contradiction is formally close to the argument developed by Broome concerning the logical incompatibility of the intuition of neutrality for

the addition of a life with the principle of personal good.[13] Note that another way to prove that the property $U1$ is incompatible with monotonicity consists of examining the constraints that that formulation of the intuition of universality imposes on the structural form of the value function $V(.)$.

The property $U1$ implies that the variation in value associated with an extra life-year, i.e., $V((a, \ldots, g) + x) - V((a, \ldots, g))$, depends neither on the quality and quantity of other life-years, nor on the quality of the added life-year. The variation in value associated with the added life-years is thus a constant. Denoting that constant by c, one can deduce, by repeated substitutions, that:

$$V((a, \ldots, g) + x) = V((a, \ldots, g)) + c$$
$$[L \text{ years} + 1] \qquad [L \text{ years}]$$
$$\leftrightarrow V((a, \ldots, g) + x) = V((a, \ldots, f)) + c + c$$
$$[L \text{ years} + 1] \qquad [L - 1 \text{ years}]$$
$$\leftrightarrow V((a, \ldots, g) + x) = (L + 1)c$$
$$[L \text{ years} + 1].$$

We thus have that the formulation $U1$ of the intuition of universality implies that the value function $V(.)$ has a simple form: it is equal to the number of life-years multiplied by a constant. Hence, the formulation $U1$ of the intuition of universality implies that only the total lifetime matters, independent from the quality of life. Thus one can see, here again, that the formulation $U1$ of the intuition of universality is not compatible with the monotonicity property.

Given the natural appeal of the monotonicity condition, the logical incompatibility of the $U1$ formulation of the intuition of universality with monotonicity is quite problematic. This suggests that this formulation of the intuition of universality is too demanding, or too strong, which leads to incompatibilities with a property as simple as monotonicity. Therefore, in the rest of this section, we will propose to depart from the $U1$ formulation of the intuition of universality, and reformulate that intuition in different ways.

As we emphasized above, the formulation $U1$ of the intuition of universality was quite strong, since it involved a triple independence of the value of the extra life-year, with respect to the quality of the added life-year, with respect to the quality of previous life-years, and with respect to the number of previous life-years. In the remaining portion of this section, we will focus on weaker formula-

13 Broome, *Weighing Lives*, 238–39.

tions of the intuition of universality, which relaxes some of those independence requirements.

Intuition of Universality (U_2):

$$V((x, \ldots, x) + x) - V((x, \ldots, x)) = V((y, \ldots, y) + y) - V((y, \ldots, y))$$

The formulation U_2 of the intuition of universality states that the value of an extra life-year whose quality is equal to the quality of previous life-years should be universal, that is, independent from the quality of (previous and added) life-years, and also independent from the initial lengths of lives (since the left-hand side and right-hand side may involve initially lives of unequal lengths). Thus the formulation U_2 involves, as the formulation U_1, a triple independence of the value of an extra life-year.

However, the conception U_2 of universality is *weaker* than the conception U_1, since in U_2 the equality of the value of an extra life-year is conditional on its quality being equal to the quality of previous life-years, unlike what prevailed under U_1. Thus U_2 states the triple independence requirement only for lives of constant quality, not for lives of nonconstant quality. This limits the scope of the triple independence, and as such makes the conception U_2 of the intuition of universality weaker than the conception U_1.

In order to understand the implications of the formulation U_2 of the intuition of universality on the form of the value function $V(.)$, it is useful to notice that if the variation in value due to the addition of a life-year of constant quality is the same on both sides of the above equation, despite the fact that the (constant) quality of life-years is not the same on the left-hand side and right-hand side, and despite the fact that the lives being compared may initially differ in terms of sizes. As a consequence, the variation in value due to the addition of a life-year must be independent from the (constant) quality of life and from the length of the initial life. Therefore, this variation must be equal to a constant. Writing that constant with the letter c, we have,

$$V((x, \ldots, x) + x) - V((x, \ldots, x)) = c.$$

Hence, we obtain, by successive substitutions:

$$\begin{gathered} V((x, \ldots, x) + x) = V((x, \ldots, x)) + c \\ [L \text{ years} + 1] \qquad [L \text{ years}] \\ \leftrightarrow V((x, \ldots, x)) = V((x, \ldots, x)) + c + c \\ [L \text{ years} + 1] \quad [L - 1 \text{ years}] \end{gathered}$$

$$\leftrightarrow V((x, \ldots, x)) = V(x) + (L-1)c$$
$$[L \text{ years} + 1] \quad [1 \text{ year}].$$

The value function $V(x, \ldots, x)$ thus takes the form of a linear combination of the value of a single life-year $V(x)$, and of the length of that life.[14] Note that this formulation of the intuition of universality leads to a value function that is increasing in the quality of the added life-year (i.e., x), and, as such, satisfies the monotonicity condition, unlike the (stronger) formulation $U1$ of the intuition of universality.

Interestingly, the United Nations HDI, in its initial form, is additive in an index of life expectancy achievements, and in an index of GDP per capita achievements (and also of an index of education achievements).[15] Hence, the initial HDI has a functional form that is compatible with the conception $U2$ of the intuition of universality, in the sense that it values the addition of a life-year with constant quality independent from the (constant) quality, and independent from the length of initial lives. The standard HDI thus captures the formulation $U2$ of the intuition of universality.

Whereas the above discussion assumes a constant quality of life profile, it may be useful to generalize that discussion to the case where the lifetime quality profile is not constant, and is, for example, equal to $(a, b, \ldots, h)$. One could then consider a value function $V(.)$ that is, as above, additive, but takes the (more general) form: $V(X) + (L-1)c$, where X denotes the *generalized average* quality of lifetime.

If the generalized average of the quality of life X is equal to the quality of the added life-year (i.e., x), then that value function satisfies the property $U2$. Note also that, provided the generalized average of the quality of life X is increasing the quality of the added life-year (i.e., x), this value function also satisfies the monotonicity condition.[16] Moreover, the value function $V(.)$ allows for a certain degree of complementarity between lifetime quantity and lifetime quality, something that was not possible under the formulation $U1$ of the intuition of universality. Clearly, adding a life-year with a quality above the average quality (i.e., $x > X$) is here regarded as good (i.e., increases value), whereas adding a life-year with a quality inferior to the average quality (i.e., $x < X$) is bad (i.e., reduces value). Finally, adding a life-year with a quality exactly equal to the average quality (i.e., $x = X$) is neutral.

Those positive results hold thanks to the fact that $U2$ is a weaker formulation

14 Indeed, it is possible to add an extra term c on the right-hand side while still respecting the condition $U2$, which leads to $V(x, \ldots, x) = V(x) + cL$.

15 See United Nations Development Programme, *Human Development Report 1990*.

16 Note that the generalized average quality is not necessarily increasing in x. One could have $X = \min\{a, b, \ldots, h, x\} = a \neq x$, or $X = \max\{a, b, \ldots, h, x\} = d \neq x$.

of the intuition of universality than U_1. It is thus compatible with monotonicity and also with some degree of complementarity, but at the cost of weakening the requirement of universality (with respect to formulation U_1).

Let us now consider an alternative formulation of the intuition of universality, which imposes not an independence of the value of the extra life-year with respect to the quality of the added life-year, but an independence only with respect to the quality of previous life-years (which may not be constant, unlike under U_2), and with respect to the number of previous life-years. This is the formulation U_3 of the intuition of universality.

Intuition of Universality (U_3):

$$V((a, b, \ldots, g) + z) - V((a, b, \ldots, g)) = V((h, i, \ldots, m) + z) - V((h, i, \ldots, m))$$

The formulation U_3 of the intuition of universality involves a double independence of the valuation of an extra life-year: (i) independence with respect to the quality of previous life-years, and (ii) independence with respect to the quantity of previous life-years (since the number of previous life-years involved on the left-hand side and right-hand side may differ).

To understand the implications of the formulation U_3 of the intuition of universality on the structure of the value function $V(.)$, let us notice that, since the variation of value due to the addition of a life-year does not depend on the quality and quantity of previous life-years, it can only depend on the quality of the added life-year, that is, on z. Hence, we have:

$$V((a, b, \ldots, g) + z) - V((a, b, \ldots, g)) = v_L(z),$$

where $v_L(z)$ is a function of z.

Hence we obtain, by repeated substitution:

$$\begin{aligned} & V((a, b, \ldots, g) + z) = V((a, b, \ldots, g)) + v_L(z) \\ & \quad [L \text{ years} + 1] \qquad [L \text{ years}] \\ \leftrightarrow\ & V((a, b, \ldots, g) + z) = V((a, b, \ldots)) + v_{L-1}(g) + v_L(z) \\ & \quad [L \text{ years} + 1] \qquad [L - 1 \text{ years}] \\ \leftrightarrow\ & V((a, b, \ldots, g) + z) = V(a) + v_2(b) + \ldots + v_{L-1}(g) + v_L(z) \\ & \quad [L \text{ years} + 1] \qquad [1 \text{ year}]. \end{aligned}$$

where $v_2(.), \ldots, v_L(.)$ are functions of the quality of each life-year.

The formulation U_3 of the intuition of universality has thus a precise implication on the structure of the value function $V(.)$. Actually, it imposes that the

value function $V(.)$ is a sum of the transformed qualities of all life-years involved, with a number of terms equal to the number of life-years in the life under study.

The functional form for $V(.)$ implied by the property $U3$ satisfies the monotonicity condition, unlike the property $U1$. Moreover, it allows also for some degree of complementarity between the quality and quantity of life. As for the formulation $U2$ of the intuition of universality, those positive results arise thanks to the fact that the property $U3$ is a weaker formulation of the intuition of universality. That more limited universality requirement allows for some compatibility with monotonicity and with the intuition of complementarity, but at the cost of weakening the universality requirement.

In sum, this section showed that the intuition of universality for the valuation of a life-year can be formulated in quite distinct ways, which all have their particular implications for the structure of the value function that measures the value of a life as a whole. Note, however, that although this section allowed us to provide precise accounts of the intuition of universality, and to explore the consequences of those formulations on the structure of value functions, this section had little to say, in general, about how priorities should be given when allocating life-years within the population. Actually, the variations in value associated with the addition of a life-year do not have direct implications in terms of priority, except if one adopts the social objective of maximizing the sum, across all individuals, of values $V(.)$.

If one adopts that particular social objective, then an interesting thing to notice is that the formulations $U1$, $U2$, and $U3$ of the intuition of universality share an important direct implication in terms of social priority: these all imply *social indifference* regarding how life-years are allocated within the population. Thus those three conceptions of the intuition of universality lead the social evaluator to be indifferent with respect to how those life-years are distributed. That result is not particularly appealing: from an egalitarian perspective, one may prefer to give social priority to individuals whose lives are of low quality or of limited quantity. This view is clearly not compatible with the formulations $U1$, $U2$, and $U3$ of the intuition of universality, at least if the goal is to maximize the sum of values $V(.)$ across individuals.

It should be stressed, however, that there is no obvious reason why the social evaluator should take, as an objective, the maximization of the sum of individual values $V(.)$. Many other social goals exist, and in those cases the above formulations of the intuition of universality do not have direct implications in terms of priority. The goal of the next section is to develop an alternative approach, in terms of social preference orderings, in order to explore, under more general social objectives, the implications of the three conceptions of the intuition of universality developed above for the allocation of life-years within a population.

2. THREE ACCOUNTS OF THE INTUITION OF UNIVERSALITY: SOCIAL PREFERENCE APPROACH

Let us now examine the implications of the intuition of universality in terms of priority in the context of a problem of life-years allocation. For that purpose, let us define an allocation as a vector of quality of life for each individual in the population, whose size is supposed to be constant and equal to N. Formally, an allocation can be written as a vector $q = (q_i)_{i=1,\ldots,N}$, where $q_i = (q_{i1}, \ldots, q_{ili})$ is the life of individual i, who enjoys a life of length l_i. We denote by Q the set of all such allocations. We denote by Q^c the subset of Q that includes all allocations with constant quality along the life. Regarding individual longevities, we will define by l the vector of individual durations of life (number of life-years). We have that $l = (l_i)_{i=1,\ldots,N}$.

Let us now reformulate the three conceptions of the intuition of universality studied in section 1 in terms of their consequences concerning the social ranking of allocations. In this section, we denote that social preordering as $\leq_s$. That social preference relation is assumed to be reflexive, transitive, and complete. As usual, strict social preference is denoted by $<_s$, whereas social indifference is written as $\sim_s$.

Throughout this section, the intuition of universality will be formulated in terms of whether adding an extra life-year to a person i is equivalent to adding an extra life-year to a person j, which is, from a formal perspective, equivalent to stating that transferring a life-year from individual j to individual i leads to social indifference. Thus, even if the formulations of the intuition of universality developed below look like properties about transfers of life-years across individuals, these are only a formal way to formulate conditions of social indifference about who receives the extra life-year.

In terms of the preordering on allocations, the formulation $U1$ of the intuition of universality states that a change in who receives an additional life-year, everything else being left unchanged, leads to an allocation that is regarded, from a social perspective, as equally good as the initial allocation, independently from the quality of the added life-year, and independently from the quantity and quality of previous life-years.

> *Intuition of Universality* ($U1$): For all q, q' in Q, if $q_{it} = q_{it}'$ for all i, t in $\{1, 2, \ldots, \min(l_i, l_i')\}$, and if there exists i, j such that: $l_i' = l_i + 1$ and $l_j' = l_j - 1$ and for all $k \neq i, j$, $l_k' = l_k$, then $q \sim_s q'$.

From the perspective of $U1$, it does not matter whether an additional life-year is given to a person with a more or less long life, or with a life of more or less high quality: changing the recipient of the extra life-year leads neither to a social im-

provement, nor to a social worsening, but is just neutral. As such, the conception $U1$ captures some idea of universality in the valuation of life-years. It states that one is socially indifferent between allocating an extra life-year to a given life or to another life.

Note that this social indifference associated with who receives the extra life-year amounts to assuming that the social valuation of life-years satisfies a triple independence: (i) independence with respect to the quality of the added life-year; (ii) independence with respect to the quality of previous life-years; and (iii) independence with respect to the quantity of previous life-years.

What are the implications of the formulation $U1$ of the intuition of universality for the allocation of life-years within a population?

Although the formulation $U1$ may seem intuitive at first glance, it has implications that are not so attractive regarding the allocation of life-years. In particular, it is incompatible with a basic monotonicity property. The monotonicity property can be stated as follows.

> *Monotonicity*: For all q, q' in $\mathcal{Q}$, if $l = l'$, if $q_{it}' > q_{it}$ for some i, t in $\{1, \ldots, l_i\}$ and $q_{it}' = q_{it}$ for all other i, t in $\{1, \ldots, l_i\}$, then $q' >_s q$.

The monotonicity property is quite weak: it states that if some allocation q' involves a higher quality of life-years for some individuals in comparison to the allocation q, everything else remaining the same in q' and q, then the allocation q' is, from a social perspective, strictly better than the allocation q.

In order to see why the formulation $U1$ of the intuition of universality is logically incompatible with the monotonicity property, let us consider two allocations q and q' satisfying the conditions described in $U1$, and let us add a third allocation, denoted by q'', which is the same as the allocation q', except that it involves a strictly higher quality of life for the extra life-year enjoyed by individual i, that is, that $q_{ili}'' > q_{ili}'$.

It is easy to see that, when comparing allocations q and q'', the formulation $U1$ of the intuition of universality implies that there must be social indifference between q and q'', for the same reasons as there is social indifference between q and q'. We thus have, by property $U1$, that:

$$q \sim_s q'' \text{ and } q \sim_s q'.$$

This implies, by transitivity, that:

$$q' \sim_s q''.$$

However, the monotonicity property requires that:

$$q'' >_s q'.$$

Thus we reach here a contradiction. That contradiction implies that the social ranking of allocations cannot satisfy both the formulation $U1$ of the intuition of universality and the monotonicity property. A choice is to be made between those properties.

Given the natural appeal of the monotonicity condition for the social ranking of allocations, this negative result supports giving up the intuition of universality, at least under its $U1$ formulation. Actually, if being universalist regarding the valuation of life-years implies violating monotonicity, and thus being socially indifferent between allocations that are clearly not equivalent at all, then the attractiveness of such a universalism can be questioned.

It should be stressed, however, that the above negative result only concerns the formulation $U1$ of the intuition of universality. As such, this cannot be generalized to all conceptions of the intuition of universality.

Let us now consider the implications of the second formulation of the intuition of universality on the allocation of life-years. Translated in terms of requirements regarding the social preference relation, the formulation $U2$ can be written as:

> *Intuition of Universality* ($U2$): For all q, q' in Q^C, if $q_{i1} = q_{i1}'$ for all i, and if there exists i, j such that: $l_i' = l_i + 1$ and $l_j' = l_j - 1$ and for all $k \neq i, j$, $l_k' = l_k$, then $q \sim_s q'$.

Thus $U2$ states that, when comparing allocations with constant quality among lives, a change in the recipient of an extra life-year from person j to person i (while keeping everything else unchanged) leads to social indifference. Whatever the durations of life for the individuals i and j, and whatever the qualities of their previous life-years, whether it is person i or person j that receives the extra life-year is neutral. Note that this conception of universality is weaker than conception $U1$, because it is here restricted to the subset of allocations in which the quality of life is constant along a given life.

In order to explore the implications of the formulation $U2$ of the intuition of universality in terms of priority, a first important step consists of examining the constraints that $U2$ imposes on the form of a social-welfare function. Actually, as shown in the appendix:

> *Characterization Theorem (Formulation U2 of the Intuition of Universality)*: A social-welfare function $W(.)$ satisfies the formulation $U2$ of the intuition of universality if and only if it takes the following form: $W(q) = F(q_i,$

$q_j, \ldots, q_N, \Sigma l_i)$, where q_i denotes the (constant) quality of life enjoyed at all life-periods by individual i under allocation q.

What is stated here is a representation result that takes the form of a logical equivalence: any social-welfare function that satisfies the property $U2$ must have that particular form, and, also, any social-welfare function that satisfies that form must also satisfy the formulation $U2$ of the intuition of universality. Interestingly, the form taken by the social-welfare function is simple: it is a function of the (constant) qualities associated with the life-years of all individuals, and also a function of the total lifetime of the population.[17]

An important corollary of this representation result is that, under the conception $U2$ of universalism, the particular distribution of life-years within the population does not matter; the only thing that matters concerning lifetime is the *total amount of life-years* that are lived. Whether the lifetime is shared more or less equally within the population does not matter.

That corollary is particularly counterintuitive. When considering the allocation of life-years within a population, a social planner may prefer, on the grounds of social justice, that life-years are distributed more equally across individual lives. Such an egalitarian perspective is incompatible with the formulation $U2$ of the intuition of universality. Being universalist under the $U2$ conception implies being socially indifferent between allocations that keep the total number of life-years constant, independently from how those life-years are distributed in the population.

To put it in different terms, the formulation $U2$ of the intuition of universality leads to being indifferent with respect to the distribution of life-years across individuals, and, as such, this is incompatible with the idea of giving priority to the poor, who can be here represented as individuals with shorter lives and lower qualities of life. The formulation $U2$ of the intuition of universality prevents giving priority to those disadvantaged individuals.

Note that this result only presupposes the formulation $U2$ of the intuition of universality, and is not based on a particular assumption concerning the way in which the social-welfare function aggregates value functions $V(.)$. Clearly, if the social ordering of allocations were based on the sum of individual value functions $V(.)$, as in section 1, one would also obtain social indifference with respect to the distribution of life-years within the population. This section provides a more general argument, according to which the formulation $U2$ of the intuition of universality leads inevitably to social indifference with respect to the distri-

17 Given that we consider populations of constant sizes, the social-welfare function $W(.)$ can also be regarded as a function of the average lifetime of the population.

bution of life-years, whatever the precise way (additive or not) in which value functions $V(.)$ enter the social objective.

That corollary of the formulation $U2$ of the intuition of universality tends to question the attractiveness of universality when formulated in that particular way. If being universalist implies being indifferent with respect to inequalities in length of life, then such a universalist perspective looks far from attractive. Ideally, we would like universalism to lead toward priority given to the disadvantaged, and, hence, toward more equality, and not to lead to indifference toward more inequality. We reach, here again, a negative result, but this negative result is relative to a particular formulation of the intuition of universality.[18]

Let us now turn to the third conception of universality developed in section 1. When reformulated in terms of its implications on the social preference ordering over allocations, the formulation U3 of the intuition of universality is defined as follows.

Intuition of Universality $(U3)$: For all q in Q, for all z in R_+, for all i, j, we have:

$$(\dots(q_{i1}, \dots, q_{ili}, z)\dots(\dots(q_{j1}, \dots, q_{jlj})\dots)$$
$$\sim_s (\dots(q_{i1}, \dots, q_{ili})\dots(\dots(q_{j1}, \dots, q_{jlj}, z)\dots).$$

The property $U3$ states that changing the recipient of an extra life-year with quality z leads to social indifference, whatever the quality and quantity of life-years lived by the possible recipients. As such, it captures some intuition of universality, in the sense that the social evaluator is indifferent between giving an extra life-year to one person or to another, whatever the lives of those persons are.

Although that conception of universality may seem appealing, it faces the same problem as the conception $U2$ studied above: by valuing transfers of life-years indifferently from the lives of the persons who are involved in the transfer (in terms of their quantity and quality), the conception $U3$ of universality goes against the idea of giving priority to the disadvantaged.

To see this, let us take a simple two-person example, involving persons i and j. The initial allocation is:

$$((q_{i1}, q_{i2}, \dots, q_{ili}), (q_{j1}, q_{j2}, \dots, q_{jlj})).$$

The formulation $U3$ of the intuition of universality states that changing the recip-

18 Note that this negative result is reached while assuming a representativity of the social preference ordering by means of a social-welfare function $W(.)$. However, as shown in the appendix, our result is actually more general, and does not necessarily require assuming the existence of such a representation.

ient of a life-year, from, let us say, person j to person i, leads to social indifference. If the last life-year of person j is reallocated to person i (with the associated quality q_{jlj}), we thus have:

$$((q_{i1}, q_{i2}, \ldots, q_{ili}), (q_{j1}, q_{j2}, \ldots, q_{jlj})) \sim_s ((q_{i1}, q_{i2}, \ldots, q_{ili}, q_{jlj}), (q_{j1}, q_{j2}, \ldots, q_{jlj-1})).$$

Then, by repeating reallocations of life-years successively, from person j to person i, one finally obtains:

$$((q_{i1}, q_{i2}, \ldots, q_{ili}), (q_{j1}, q_{j2}, \ldots, q_{jlj})) \sim_s ((q_{i1}, q_{i2}, \ldots, q_{ili}, q_{jlj}, \ldots, q_{j2}), (q_{j1})).$$

Thus property U_3 leads to social indifference between two allocations that are extremely different: whereas in the initial allocation, the lifetime is divided between persons i and j, in the final allocation, almost the entire lifetime is concentrated on person i, whereas only a single life-year remains for person j. That highly unequal distribution of lifetime does not seem to be as socially desirable as the initial allocation, but this is what formulation U_3 of the intuition of universality implies. Repeated use of the universality property U_3 leads to social indifference between allocations that are characterized by quite different degrees of inequality in the distribution of lifetime among persons.

From the perspective of social justice, one would prefer, on the contrary, to give priority in the allocation of life-years to disadvantaged individuals, who have either shorter lives or lives of worse quality. The intuition of universality is hardly compatible with giving priority to the disadvantaged. On the contrary, it leads to social indifference with respect to how life-years are allocated between persons. Here again, as for the formulation U_2, the intuition of universality goes against this ideal of giving priority to the disadvantaged.

Note also that, as for the conception U_2, the argument provided here does not rely on a particular functional form for the social objective. Obviously, if the social goal is, as in section 1, to maximize the sum of value functions $V(.)$, then we would also obtain social indifference with respect to who receives the extra life-year. But the argument developed here is more general, since this does not presuppose any particular social objective—that is, the social ordering does not need to be based on the mere sum of value functions $V(.)$. Thus, we reach a robust result on the conflict of conception U_3 of universality with giving priority to the disadvantaged.

In sum, this section leads to quite negative results concerning the implications of the intuition of universality. We showed that either the intuition of universality is incompatible with the monotonicity property (conception U_1), or leads to social indifference with respect to how life-years are allocated within the

population, which goes against the ideal of giving priority to the disadvantaged (conceptions U_2 and U_3).

Whereas this section reached some negative results concerning the intuition of universality, one may wonder whether abandoning that intuition in favor of the intuition of complementarity would allow obtaining more appealing implications. In particular, one may be curious to see whether adopting the intuition of complementarity would allow better meeting of the ideal of giving priority to the disadvantaged. That question is explored in the next section.

3. THE INTUITION OF COMPLEMENTARITY AND PRIORITY TO THE WORST-OFF

Under the intuition of complementarity, the value of a life-year depends on what that life-year allows—that is, on the quality of that life-year. At first glance, one may believe that the intuition of complementarity, by leading to assigning a higher value to life-years characterized by a higher quality (unlike the intuition of universality), could favor the allocation of life-years toward more life-years given to individuals who enjoy a high quality of life.

But that belief is actually wrong: when allocating life-years, the valorization of those years is only *one* aspect of the problem. Another crucial aspect concerns the priority that the social evaluator assigns to the well-being levels of the different individuals, and, in particular, their aversion to inequality.[19] When the aversion to inequality is large, it can offset the valorization dimension, and lead to assigning more life-years to individuals with low life quality. It is actually quite simple to combine the intuition of complementarity with giving priority to the disadvantaged.

To see that, let us assume that the value of an individual life takes a standard, time-additive form, that is:

$$V(q_i) = \sum_{t=i}^{l_i} u_i(q_{it}),$$

where $u_i(q_{it})$ represents the temporal utility associated with the life-year t for individual i.

For the sake of simplicity, we will assume here that lifetime is continuous rather than discrete, and thus consider the equivalent form in continuous time:

$$V(q_i) = \int_t^{l_i} u_i(q_{it})\, dt.$$

The social-welfare function takes the general form:

19 On the assignment of social priority, see Adler, *Well-Being and Fair Distribution* and *Measuring Social Welfare*.

$$W(q) = W(V(q_1), \ldots, V(q_N)).$$

Within that framework, the marginal social-welfare from increasing the duration of the life of individual i is given by the derivative:

$$\partial W/\partial l_i = (\partial W/\partial V(q_i))\, u_i(q_{il_i}).$$

The left-hand side of that equation is the variation in social welfare associated to a minor change in the duration of the life of individual *i*. This variation is equal to the product of two factors.

First, it depends on the degree of priority of the individual from a social perspective, which is captured by the factor $(\partial W/\partial V(q_i))$. This degree of priority clearly depends on the degree of inequality aversion exhibited by the social-welfare function. If person *i* is particularly disadvantaged, an inequality-averse social planner assigns a high weight to improving the well-being of that person. This first effect is the social-weighting effect.

Second, the marginal social welfare associated with a change in duration of the life of individual *i* depends also on the value of this extra life-year for the individual, based on the quantity and quality of their past life, and also based on the quality of the extra life-year itself. One can expect that a life profile with a higher quality of life will generally imply a higher value for an increase in the duration of life. That second effect is the individual valuation effect (which may depend on individual subjective preferences or some other objective approach to the valuation of individual lives).

In the case of increasing the lifetime of an individual whose life has low quality, the social-weighting effect and the individual-valuation effect go in opposite directions when the social planner is inequality averse. In that case, the social-weighting effect is strong, while the individual-valuation effect is low. On the contrary, when considering the marginal social welfare from increasing the duration of life of a person whose life has high quality, the opposite arises: the individual-valuation effect is large, while the social-weighting effect is low.

At the end of the day, whether a higher marginal social value is assigned to increasing the length of life of the person with a low or high life quality depends on the degree of inequality aversion exhibited by the social-welfare function, and also on individuals' valuations of life. It is quite possible that a higher marginal social value is assigned to increasing the duration of life of a person with a low life quality, despite the individual valuation effect. This is definitely the case when the social-welfare function exhibits a high degree of inequality aversion.

To show this, let us take a simple analytical example, where the function $u_i(q_{it})$ takes the following form:

$$u_i(q_{it}) = [{q_{it}}^{1-a} - {q_0}^{1-a}]/(1-a).$$

Moreover, let us suppose that the social-welfare function takes a standard Atkinson form:

$$W = \Sigma(V(q_i))^{1-e}/(1-e),$$

where the parameter e captures the sensitivity to inequalities in well-being across individuals.

In that analytical example, and supposing a constant quality of life $q_{it} = q^i$, the marginal social welfare from increasing the duration of life of person *i* is equal to:

$$\partial W/\partial l_i = (l_i u_i(q^i))^{-e} u_i(q^i).$$

Note that, when the ethical parameter *e* equals 0, the marginal social welfare from increasing the duration of life of individual *i* is equal to $u_i(q^i)$, and, hence, is increasing with the quality of life enjoyed by person *i*. In that case, the social-weighting effect is dominated by the individual-valuation effect, and so a larger priority is given to individuals with a higher quality of life.

But that is not the only possible case. Actually, under a large interval of values for the ethical parameter *e*, the opposite will take place, and the social-weighting effect will dominate the individual-valuation effect, leading to priority to the disadvantaged individuals.

It is straightforward to see that, when the ethical parameter *e* equals 1, the marginal social welfare from increasing the duration of life of individual *i* is equal to merely $1/l_i$—that is, to the inverse of person *i*'s duration of life. Hence, in that case, a higher priority will be given to individuals with a short life, and a lower priority to individuals with a longer life.

Alternatively, when *e* is superior to 1, an even larger priority is given to the disadvantaged individuals, since the marginal social welfare from increasing the length of life of a person *i* is then not only decreasing with the duration of life of that person, but also decreasing with the quality of life. Thus priority is here given to individuals with shorter lives and lives of lower quality.

Those examples suffice to illustrate that it is possible—and actually quite easy—to accommodate the intuition of complementarity with the ideal of giving priority to the disadvantaged. The intuition is that the marginal social value of increasing the duration of life of a person depends not only on the quality of that life (through the individual-valuation effect), but also on the weight that is given to improving the situation of that person in the social-welfare function (the social-weighting effect). When the latter dominates the former, priority is given to individuals with a shorter life and with a lower quality of life.

4. CONCLUDING REMARKS

Given that various policies—health policies, safety policies, development policies—influence mortality, and, hence, individual lifetimes, the valuation of life-years has become a necessary stage in the design of optimal policies. The definition of optimal policies in life-affecting domains requires governments to be able not only to weight life-years against resources, but, also, life-years enjoyed by some persons against life-years enjoyed by other persons. Moreover, at the descriptive level, the measurement of economic development requires the ability to weight achievements in terms of longevity in comparison to achievements on other dimensions of life, and, also, to make longevity achievements in some countries comparable with longevity achievements in other countries.

When considering the valuation of life-years, two basic intuitions arise: on the one hand, the intuition of universality, according to which the value of a life-year should be universal, and, hence, independent from the duration and the quality of lives considered, and, on the other hand, the intuition of complementarity, according to which the value of a life-year should depend on what that life-year allows for, and, hence, on its quality.

Those two intuitions are plausible, but hardly compatible: the intuition of universality requires that the value of a life-year is universal, and, hence, does not depend on its quality, which goes against the intuition of complementarity, which makes the valuation of a life-year dependent on its quality. Thus a choice is to be made between those two intuitions concerning the valuation of life-years.

In order to cast original light on that ethical dilemma, this paper proposes to provide several distinct accounts of the intuition of universality, and to explore their logical implications in terms of the valuation of life-years, and, also, in terms of the priority to be given to the disadvantaged when considering the allocation of lifetime within a population.

Our results suggest that the intuition of universality, whatever the precise formulation considered, leads to implications that are far from appealing. Our accounts of the intuition of universality lead either to a conflict with a basic principle of monotonicity (i.e., the conception U_1), or lead to a conflict with giving priority to the disadvantaged (i.e., conceptions U_2 and U_3). Those conflicts are quite problematic: imposing a universal valuation of life-years would lead to social indifference with respect to the distribution of lifetime within the population. Such social indifference would go against the ideal of equality, and, as such, is counterintuitive and hard to justify.

On the contrary, the intuition of complementarity can be compatible with the idea of giving priority to the disadvantaged, and, as such, does not imply a

social indifference with respect to how life-years are distributed within the population, unlike the intuition of universality. The underlying intuition is that the dependence of the valuation of a life-year on quality of life is only one aspect of the social valuation of life-years, which depends also on how individual interests are weighted in the social-welfare function. It is thus possible, when the social-welfare function exhibits a sufficiently high degree of inequality aversion, to conciliate the intuition of complementarity with the ideal of giving priority to individuals with low qualities of life.

All in all, this paper suggests that the intuition of universality, although it may seem appealing at first glance, leads, at the end of the day, to the opposite of what it aims at: by valuing all life-years in a uniform way, the intuition of universality is not compatible with giving priority to the disadvantaged, and, hence, tends to play against equality. On the contrary, the intuition of complementarity can be made compatible with the ideal of giving priority to the disadvantaged, and, hence, is more compatible with equality.

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APPENDIX

The characterization result takes the form of an equivalence between a social-welfare function $W(q)$ satisfying property $U2$ and a social-welfare function taking the form $W(q) = F(q_i, q_j, \ldots, q_N, \Sigma l_i)$.

To prove that equivalence result, we proceed in two steps.

Let us first prove that a social-welfare function taking the form $W(q) = F(q_i, q_j, \ldots, q_N, \Sigma l_i)$ satisfies the property $U2$.

To see this, let us take a three-person case, with (constant) qualities of life a, b, and c, and durations of life m, n, and o. Let us denote by $a.m$ a life of m years with constant quality a. The allocation q is thus written $(a.m, b.n, c.o)$. Let us now compare that allocation with another allocation, q', where a life-year is transferred from the second person to the first person. The allocation q' is thus written as $(a.(m+1), b.(n-1), c.o)$.

The property $U2$ requires that there is social indifference between allocations q and q', that is: $W(a.m, b.n, c.o) = W(a(m+1), b(n-1), c.o)$.

It is easy to see that this equality is satisfied by any function taking the form

$W(q) = F(q_i, q_j, \ldots, q_N, \Sigma l_i)$. Indeed, in our three-person case, the function takes the form: $W(a.m, b.n, c.o) = F(a, b, c, m + n + o)$. It does satisfy the equality mentioned above, since transfer of a life-year maintains the total number of life-years unchanged. Indeed, we have:

$$W(q) = F(a, b, c, m + n + o) = F(a, b, c, m + 1 + n - 1 + o) = W(q')$$

as required by property *U*2.

The same argument could be formulated for any case with $N > 3$, with any transfer of life-years. Thus we have that any social-welfare function taking the form $W(q) = F(q_i, q_j, \ldots, q_N, \Sigma l_i)$ satisfies the property *U*2.

Let us now prove that any social-welfare function satisfying the property *U*2 takes the form $W(q) = F(q_i, q_j, \ldots, q_N, \Sigma l_i)$. To prove this, let us turn back to our three-person case. We have, by repeated use of the property *U*2:

$$\begin{aligned} W(a.m, b.n, c.o) &= W(a.(m + 1), b.(n - 1), c.o) \\ &= W(a.(m + 2), b.(n - 1), c.(o - 1)) \\ &= W(a.(m + 3), b.(n - 2), c.(o - 1)) \\ &= \ldots \\ &= W(a.(m + n - 1 + o - 1), b.1, c.1) \\ &= W(a.(l_1 + l_2 + l_3 - (3 - 1)), b.1, c.1). \end{aligned}$$

Since the population size $N = 3$ is a constant, we thus have:

$$W(a.m, b.n, c.o) = F(a, b, c, l_1 + l_2 + l_3).$$

That is, the social-welfare function takes the form $W(q) = F(q_i, q_j, \ldots, q_N, \Sigma l_i)$. A similar proof could be provided for any $N > 3$.

Finally, it should be stressed that, whereas the above proof assumes the existence of a representation of the social ordering $\leq_s$, such an assumption is not necessary for the purpose at hand. Actually, it can be shown that there exists another preorder $\leq^*$ defined on $((q_i)_i, \Sigma l_i)$, which is such that:

$$((qi)i, \Sigma li) \geq^* ((qi')i, \Sigma li')$$

$$\leftrightarrow$$

$$(q1.(\Sigma li - N + 1), qi.1) \geq (q1'.(\Sigma li' - N + 1), qi'.1).$$

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