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WEAK INDEPENDENCE IS SUFFICIENT FOR MAKING WEAK SUPERIORITY COLLAPSE INTO STRONG SUPERIORITY

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THIS NOTE complements the seminal work of Gustaf Arrhenius and Wlodek Rabinowicz, who developed a formal approach to Millian superiorities, which concern the aggregation of goods of different qualities.[1] What I present in this article appears to have been accidentally overlooked in the literature. Given that the work by Arrhenius and Rabinowicz has been extensively studied and extended by many scholars, I believe this oversight warrants formal examination.[2]

I start by introducing the two concepts of Millian superiority, strong superiority and weak superiority, which trace their origins back to James Griffin's work.[3] The distinction between these two concepts is significant in population ethics and in value theory. If the quality of one object is very high while that of another is very low, is it the case that as long as there is a sufficiently large amount of the former, no quantity of the latter—no matter how great—can ever outweigh it? Griffin emphasizes the importance of such cases by using the label 'discontinuity'.[4] This type of case—explicitly formulated in terms of weak superiority—has been considered a possible response to Derek Parfit's "repugnant conclusion," according to which, for any population with very high levels of welfare, there exists a much larger population whose members have lives barely worth living that is nevertheless better.[5]

If weak superiority holds, a population of a certain size with very high welfare cannot be worse than any number of lives barely worth living; in other words, quality can outweigh quantity under such a welfare difference. This avoids the repugnant conclusion. However, even if we accept this as a solution,

1 Arrhenius and Rabinowicz, "Millian Superiorities."

2 Arrhenius and Rabinowicz, "Millian Superiorities."

3 Griffin, *Well-Being*, 83–89.

4 Griffin, *Well-Being*, 83–89.

5 Parfit, *Reasons and Persons*, 388.

it does not follow that a single person living a wonderful life is better than a large number of people whose lives are barely worth living, no matter how numerous they may be. If this view is taken, one is led to accept weak superiority while rejecting strong superiority.

Karsten Klint Jensen showed that these two concepts coincide under a strong form of independence (Independence), a separability axiom.[6] However, as this article demonstrates, the distinction between the two notions disappears under a logically weaker version, Weak Independence. In other words, even the more permissive separability assumption suffices to force weak superiority to collapse into strong superiority. Arguing for weak superiority while rejecting strong superiority is more difficult than it might initially appear.

1. THE COLLAPSE RESULT

We consider a given set of objects as the domain. Objects are denoted by e, e', e'', and so on. We assume that the domain is closed under the concatenation of objects. An at-least-as-good-as relation is defined on this domain, and the associated betterness relation is induced from it. That is, if e and e' belong to the domain, then so does (e, e'). This implies that for any finite number n, if e belongs to the domain, then so does ne, where ne denotes an object consisting of n copies of e. An ordered list of objects is called a sequence of objects, typically denoted by $(e_1, \ldots, e_n)$. A sequence $(e_1, \ldots, e_n)$ of objects is called decreasing if and only if each object is better than its immediate successor, i.e., e_i is better than e_{i+1} for all $i = 1, \ldots, n-1$.

Now let us introduce the concept of strong superiority. This is the case where, no matter how many copies of e' are combined, they never become better than e. This can be formally stated as follows.

> *Definition 1*: An object e is strongly superior to an object e' if and only if for all positive integers n, e is better than ne'.

The next concept is weak superiority. This concerns the case where there exists an m such that me becomes strongly superior to e'. Its formal statement is as follows.

> *Definition 2*: An object e is weakly superior to an object e' if and only if for some positive integer m and all positive integers n, me is better than ne'.

6 Jensen, "Millian Superiorities and the Repugnant Conclusion," 283–84.

It is easy to see that strong superiority logically implies weak superiority. However, the converse is not true.[7] In other words, weak superiority does not necessarily collapse into strong superiority. Arrhenius and Rabinowicz show the following.

> *Observation 1*: Consider any two objects e and e' such that e is better than e'. If e is weakly superior to e', without being strongly superior to it, then the domain must contain a finite decreasing sequence of objects in which the first element is strongly superior to the last one, but no element is strongly superior to its immediate successor.[8]

This observation challenges a widespread intuition. As Arrhenius and Rabinowicz note, "many people seem to have a strong intuition that a decreasing sequence in which the first element is strongly superior to the last one must contain an element that is strongly superior to its immediate successor."[9] Notably, Observation 1 not only serves as a counterexample to this intuition but also demonstrates how misunderstandings can arise from the subtle distinction between weak and strong superiority.

As a next step, Arrhenius and Rabinowicz show how the widespread intuition can be correct. They impose a sort of separability axiom.

> *Weak Independence*: If an object e is at least as good as e', then replacing e' by e in any whole results in a whole that is at least as good.

Under this axiom, the intuition that many seem to hold turns out to be correct as long as transitivity and completeness are assumed. This is Observation 2 of Arrhenius and Rabinowicz, which is stated as follows.

> *Observation 2*: Suppose that the first element in a sequence $e_1, \ldots, e_n$ is strongly superior to the last one. If 'is at least as good as' is a complete and transitive relation, Weak Independence implies that some element in the sequence is strongly superior to its immediate successor.[10]

As we have seen, the main aim of Arrhenius and Rabinowicz is to examine a finite sequence where the first object is strongly superior to the last one.[11] However, their arguments also demonstrate that the gap between weak superiority

7 If e is strongly superior to e', then e is better than e'. However, e is not necessarily better than e' even if e is weakly superior to e'.

8 Arrhenius and Rabinowicz, "Millian Superiorities," 133, and "Value Superiority," 239. In this article, I use the statements in their later paper.

9 Arrhenius and Rabinowicz, "Value Superiority," 234.

10 Arrhenius and Rabinowicz, "Millian Superiorities," 134, and "Value Superiority," 239–40.

11 Arrhenius and Rabinowicz, "Millian Superiorities."

and strong superiority is the key. Jensen directly examines when the two concepts coincide.[12] He uses a stronger axiom of independence, replacing 'if' with 'if and only if'.

> *Independence*: An object *e* is at least as good as *e′* if and only if replacing *e′* by *e* in any whole results in a whole that is at least as good.

Jensen proves the following, which is stated as Observation 3 in a later article by Arrhenius and Rabinowicz.

> *Jensen's Observation*: Assume that 'is at least as good as' is a complete and transitive relation. If Independence is satisfied, then weak superiority implies strong superiority.[13]

Under Independence, weak superiority collapses into strong superiority. In other words, there is no gap between weak superiority and strong superiority. This is critical for Jensen, who scrutinizes Griffin's argument concerning weak superiority, which Griffin calls *discontinuity*.[14] More precisely, Griffin's proposed solution to the repugnant conclusion relies on the idea of weak superiority where enough of a superior value outranks any amount of an inferior value, while still allowing smaller amounts of the superior value to be outweighed by large amounts of the inferior value. However, Griffin mistakenly assumes this can work in an additive value framework, where the value of combinations is simply the sum of individual values. Jensen proves (in this observation) that this is impossible—under Independence or additivity, if any multiple of a value is superior to another value, then even a single unit must be superior.

Hence, Griffin's proposed solution fails because additive value is incompatible with the gap between weak and strong superiority that is required by his solution. In the presence of additive value, weak superiority collapses into strong superiority, and thus, the distinction on which his proposal depends disappears.

Despite using a stronger version of independence in his definition, Jensen does not seem to explicitly consider whether this stronger version is necessary for his results. He simply notes in a footnote that "Arrhenius and Rabinowicz only assume the 'only if'—part in their Independence condition—that is all they need for their Observation 2."[15] Moreover, the later article of Arrhenius and

12 Jensen, "Millian Superiorities and the Repugnant Conclusion," 283–84.

13 Jensen, "Millian Superiorities and the Repugnant Conclusion," 283–84; and Arrhenius and Rabinowicz, "Value Superiority," 239–41.

14 Griffin, *Well-Being*, 85.

15 Jensen, "Millian Superiorities and the Repugnant Conclusion," 282.

Rabinowicz seemingly assumes that the strengthening of Weak Independence to Independence could be necessary for deriving Jensen's Observation.[16] This is because Arrhenius and Rabinowicz introduce both Weak Independence and Independence and prove Jensen's Observation by using the latter axiom.[17]

However, the following is correct.

> *Observation*: Assume that 'is at least as good as' is a complete and transitive relation. If Weak Independence is satisfied, then weak superiority implies strong superiority.

Assume that e is weakly superior to e', but e is not strongly superior to e'. Since e is weakly superior to e', there exists a natural number m such that me is better than ne' for all natural numbers n. Since e is not strongly superior to e', there exists a natural number k such that ke' is at least as good as e. By Weak Independence, (ke', e) is at least as good as $2e$. Again, since ke' is at least as good as e, Weak Independence implies that $2ke'$ is at least as good as (ke', e). Hence, transitivity implies that $2ke'$ is at least as good as $2e$.

Since $2ke'$ is at least as good as $2e$, Weak Independence implies that $(2ke', e)$ is at least as good as $3e$. Since ke' is at least as good as e, Weak Independence implies that $3ke'$ is at least as good as $(2ke', e)$. Transitivity implies that $3ke'$ is at least as good as $3e$. By repeating this, we establish that mke' is at least as good as me. This contradicts the fact that me is better than ne' for all natural numbers n. This establishes the claim.

I established that Jensen's Observation holds under Weak Independence rather than full Independence. The gap between weak superiority and strong superiority, which is significant in population ethics and in Griffin's problem, disappears under Weak Independence. Notably, there is a gap between Independence and Weak Independence.[18] When even this central distinction

16 Arrhenius and Rabinowicz, "Value Superiority."

17 Arrhenius and Rabinowicz, "Value Superiority."

18 Under Independence, the following is correct.

> *Observation*: If Independence is satisfied, then, for all e, e', e'', (e', e) is indifferent to e' if and only if (e'', e) is indifferent to e''. In other words, adding e is indifferent independently of the objects to which e is added, whenever adding e to some object is indifferent.

The proof is simple. Now, we assume that (e', e) is indifferent to e'. Independence implies that (e'', e', e) is indifferent to (e'', e'). Again, Independence implies that (e'', e) is indifferent to e''. The opposite direction is proved analogously. Notably, this observation does not rely on either transitivity or completeness.

This observation does not hold under Weak Independence. Now, I offer a counterexample. Take two objects e^* and $\hat{e}$. Consider the following relation:

1. $(e, \hat{e})$ is indifferent to e if e includes e^*;

between the two Millian concepts collapses, the scope of the theory is significantly restricted.

2. Implications for Millian Value Theory

To illustrate the significance of the equivalence between the two Millian concepts established here, recall Observation 2 of Arrhenius and Rabinowicz, which states, roughly, that in any sequence whose first element is strongly superior to its last element, there must exist some adjacent pair at which strong superiority holds.[19] This observation implies that if e is strongly superior to e', then for any e'', e is strongly superior to e'' or e'' is strongly superior to e'. This suggests that objects are separated by the strong superiority relation.

The equivalence result established in this article implies that even if e_1 is only weakly superior to e_n, there must exist some e_i (for $i = 1, \dots, n-1$) that is strongly superior to its immediate successor. In other words, we obtain the following.

> *Observation 2′*: Suppose that the first element in a sequence $e_1, \dots, e_n$ is weakly superior to the last one. If 'is at least as good as' is a complete and transitive relation, Weak Independence implies that some element in the sequence is strongly superior to its immediate successor.

In other words, in any sequence whose first element is weakly superior to its last element, there must exist some adjacent pair at which strong superiority holds. This strengthens Observation 2.

Finally, a question that warrants further investigation is whether certain Millian concepts can be retained without committing to strong superiority. One possible way is to refrain from imposing even Weak Independence. Another is to weaken Millian superiority further, beyond Weak Superiority. For example, one might consider a requirement such as: "If an object e is at least as good as e', then replacing e' with e in any whole yields a whole that is not worse." This condition may be termed *Nondeterioration*.[20] Exploring such logically weaker concepts of Millian superiority could substantially expand the range of possible

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2. Noncomparability holds for all other cases.

It is easy to check that the aforementioned observation fails. Note that this relation satisfies Weak Independence, while it violates Independence. This implies that there is a fundamental logical gap between Weak Independence and Independence. Thus, the fact that only Weak Independence is necessary is a noteworthy point.

19 Arrhenius and Rabinowicz, "Millian Superiorities," 134.

20 This condition was suggested by an anonymous reviewer.

theories. This also suggests the possibility of dropping completeness. These issues are left for future research.[21]

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